S282 Astronomy

Stellar distance and motion

Study time: 90 minutes

Summary

In this exercise you will be using measurements of parallax and proper motion from the European Space Agency (ESA) Hipparcos satellite to calculate the distance and speed of a number of stars along with the uncertainties in these quantities. You will construct a spreadsheet to carry out the necessary calculations. This activity is related to material in Sections 3.2.1 and 3.2.2 of *An Introduction to the Sun and Stars*.

Learning outcomes

- Calculate the distance and transverse speed from measured parallax and proper motion.
- Understand the concepts of absolute and relative uncertainties and the ability to determine them in derived quantities.
- Use a spreadsheet to perform calculations on astronomical data.

Background to the activity

Although the familiar pattern of the constellations appears unchanging, the stars are not stationary in space. On a large scale stars move with the rotation of the galaxy, but on a local scale stars are also moving through space with respect to the Sun. Over a long period of time nearby stars will appear to move compared to the background of distant stars. Figure 1 (*overleaf*) shows the appearance of the constellation Ursa Major over a period of time. This movement is known as *proper motion* and is measured in arc seconds per year.

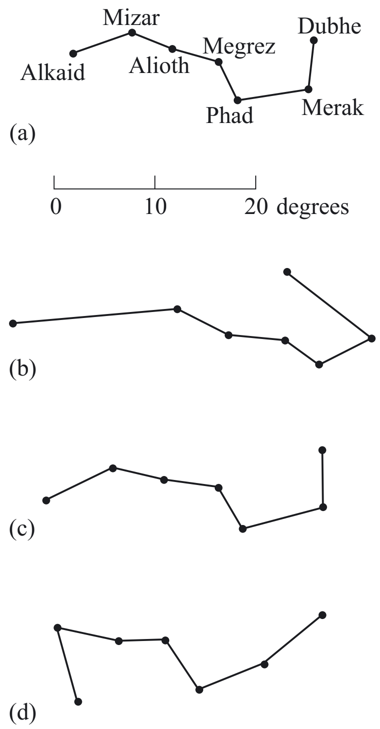


Figure 1 (a) The Plough; part of a larger constellation called The Great Bear (Ursa Major) as it appears today, and (b) 100 000 years ago, (c) 5000 years ago, and (d) 100 000 years in the future. (This is Figure 3.2 in *An Introduction to the Sun and Stars*.)

Of course, a star will only appear to move if it has a component of motion across our line of sight: a star that was moving directly towards or away from the Sun would not appear to move. In general, the actual movement through space is at an angle to our line of sight, giving a combination of transverse and radial motion as shown in Figure 2, so the total motion through space depends on both the transverse and radial speeds.

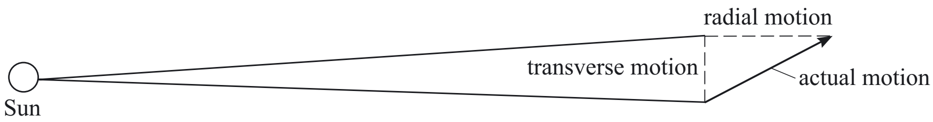


Figure 2 The actual motion of a star through space is a combination of the transverse motion and radial motion.

Typical speeds of motion can range up to over a hundred kilometres per second.

It is not possible to measure this motion directly – as with so many things in astronomy it is necessary to piece together information from a number of different sources. In this case these sources are:

- motion against background stars (proper motion)
- parallax measurements to get distance
- measurement of Doppler shift to give radial speeds.

Measurements

As mentioned above, the movement of stars against the background of distant stars is known as proper motion. It is an angular measurement: the actual speed of the motion through space will depend also on the distance of the star from the solar system. A fast moving star that is far from the Sun will show only a small proper motion, whereas a closer star moving at the same speed would cover a larger angle of view in the same time and therefore have a larger proper motion.

As discussed in *An Introduction to the Sun and Stars* Section 3.2.1, the distance to nearby stars can be measured using the method of parallax. As the Earth orbits the Sun the changing point of view causes nearby stars to appear to move across the sky relative to more distant stars in the background. Over the course of a year the star will appear to describe a parallactic ellipse. Given the fixed baseline of the Earth's orbit the size of this parallax movement depends only on the distance of the star: a more distant star will show a smaller parallax movement than one nearby.

The most convenient unit of measurement for stellar distances is the parsec. (See Figure 3.7 in *An Introduction to the Sun and Stars*.)

For a star with a parallax of p arc seconds the distance (in parsecs) is given by:

$$d/\mathrm{pc} = \frac{1}{p/\mathrm{arcsec}}$$ (*An Introduction to the Sun and Stars*, Equation 3.7)

By combining the distance measured by parallax and the (angular) proper motion the actual transverse velocity can be found.

In addition to moving across our line of sight (transverse motion) the intrinsic velocity of a star may also have a component towards or away from the Earth (radial motion). Motion along a line of sight cannot be detected as movement against the background stars, but it can be measured instead by observing the Doppler shift of known lines in the star's spectrum.

In this activity you will use data from the Hipparcos mission, which measured the positions of many stars without the blurring introduced by the presence of the Earth's atmosphere. Since Hipparcos was designed to measure positions of stars and not their spectra it provided parallax and proper motions but not radial velocities. You will use these data to investigate the distances and transverse motions of a few of these stars.

The data

Table 1 contains data for a selection of nearby stars obtained from the ESA Hipparcos catalogue (http://www.rssd.esa.int/Hipparcos/).

The uncertainties in the Hipparcos data can be regarded as ±1 milli-arcsec for the parallax and ±1 milli-arcsec yr^{-1} for the proper motion.

Table 1 Stellar motion data for a selection of nearby stars.

Star name	Parallax /milli-arcsec	Proper motion /milli-arcsec yr^{-1}
Alpha CMa (Sirius)	379.2	1339.4
Alpha Ori (Betelgeuse)	7.6	29.4
Beta Ori (Rigel)	4.2	2.0
Alpha Tau (Aldebaran)	50.1	199.5
Alpha UMa (Dubhe)	26.4	140.9
Kapteyn's star	255.3	8670.5

The activity

The aim of the activity is to calculate the distance and transverse speed of all seven stars using the data supplied. Furthermore you will also calculate the uncertainties in these two quantities. Of course you could do all the required calculations using a calculator: a reasonable prospect for data from one star, but less appealing for a larger data set. This is why a spreadsheet is used here – it provides a way of repeating the same type of calculation many times over. Although you might have to spend some time learning how to use a spreadsheet, by the end of this activity you should be able to appreciate its potential for helping you to carry out calculations. The *Using Spreadsheets* guide on the course website gives additional help.

Set up spreadsheet

Create a new sheet

If you have not already done so, start StarOffice™. The first thing to do is to create a new spreadsheet to hold the data and calculations.

From the main StarOffice menu select File | New | Spreadsheet. This creates a new, blank sheet.

Data (parallax and proper motion) for seven stars are given in Table 1. We want to calculate the distance and proper motion for each of the stars along with the uncertainties in both these quantities. A sensible way to set out the spreadsheet would be in seven columns, headed: 'Star name', 'Parallax', 'Distance', 'Uncertainty in distance', 'Proper motion', 'Transverse speed', 'Uncertainty in transverse speed'.

Enter data

Now enter the data given in Table 1 into the appropriate columns in the sheet, leaving blank columns for the results. Be sure to label the rows and columns clearly. In particular spreadsheets are good at manipulating numbers, but they do not keep track of units very well so make sure that you have included the units in the column heading labels.

Save sheet

Once you have started entering data it is a good idea to save the spreadsheet. Since this is a new spreadsheet you need to choose a name and, most importantly, a location for the spreadsheet file.

(If you have not already done so, it is a good idea to create a separate folder to hold all of your S282 work, to make it easier to find later on.

From the StarOffice menu, select File | Save as.... Enter a meaningful name for your sheet, and select the folder where you want to store the document. Press Save when you are ready.

As you continue to work through this activity, remember to save your work regularly.

Formatting and labelling

As in the earlier activities, you should consider carefully the layout of your spreadsheet. In addition to the data you should include sufficient labelling and other information to allow the user of the spreadsheet to understand what is going on. (You may wish to refer back to the formatting that you did if you completed the earlier activity on sunspot numbers.)

As a minimum, you should include the following information on your sheet:

- title
- reference to source of data (including website address (URL) if appropriate)
- explanatory comments and notes.

You might also want to consider using shading to indicate which cells contain input data and which ones contain the results of calculations (for example, cells containing data and constants could have a light yellow background, and those containing results of calculations a darker yellow). However, don't make it too lurid and give some thought to what the sheet might look like if printed out in monochrome.

Feel free to experiment with different formats, but try to maintain a consistent style as you create more sheets.

At this stage you should have a sheet that resembles the one shown in Figure 3. (It may be useful to ensure that your sheet uses the same row numbers and column letters as the example shown in Figure 3, since the explanatory notes below use cell references that relate to this spreadsheet.)

	A	B	C	D	E	F	G
1	**S282 Activity – Stellar distance and motion**						
2	**Author:** my name	**Last updated:** today's date					
3	**Source of data: http://astro.estec.esa.nl/Hipparcos**						
4							
5	parsec	3.09E+013 km					
6	milli-arcsec yr^-1	1.54E-016 rad s^-1					
7							
8	**Star name**	**Parallax**	**Distance**	+/-	**Proper motion**	**Transverse speed**	+/-
9		milli-arcsec	parsec		milli-arcsec yr^-1	km s^-1	
10	Alpha CMa (Sirius)	379.2			1339.4		
11	Alpha Ori (Betelgeuse)	7.6			29.4		
12	Beta Ori (Rigel)	4.2			2		
13	Alpha Tau (Aldebaran)	50.1			199.5		
14	Alpha UMa (Dubhe)	26.4			140.9		
15	Kapteyn's star	255.3			8670.5		
16							

Figure 3 The spreadsheet prior to carrying out any calculations.

If you are not familiar with spreadsheet formatting you may find difficulty in making an exact copy of the spreadsheet below because StarOffice re-formats your typing. If you want to learn how to turn off this reformatting see Appendix A, at the end of these instructions.

Calculate distance

Once the data have been entered you need to enter a suitable formula to calculate the distance. You should decide for yourself what formula to enter. (Think about what calculation you need to do based on Equation 3.7 given above.)

Units

As mentioned earlier, spreadsheets do not know about units. You must therefore think very carefully about units and account for them yourself in any formulae you enter.

■ Here, the parallax values are given in milli-arcsec. What must you do in your formula to account for this?

❑ You must explicitly divide the parallax values by 1000 to convert them from milli-arcsec to arc seconds. If you use a value more than once in a formula, you must convert the units every time they are used.

Formulae in a spreadsheet always start with an '=' sign. Click on the first blank cell in the Distance column of your table (in the spreadsheet shown in Figure 3, this is cell C10), and type '=' followed by the formula you have chosen.

(Consult the *Using Spreadsheets* guide for assistance on entering formulae.)

Checking

This is *very* important: as you have probably just found out, entering formulae can be a little tricky at first. You should *always* check the results of your calculations before proceeding.

How could you check that the formula you have just entered is correct? Well, one way would be to do the calculation by hand or on your calculator to check the result. Another method is to try it out on some data where you know the answer already. For example, a parallax of 1000 milli-arcsec should give a result of 1 parsec, and 100 milli-arcsec should give 10 parsecs (always check at least a couple of numbers to make sure you haven't got the right answer by accident!). You could also look up the actual known distance to one of the stars in Table 1 and check your calculation that way.

If you are having difficulty in entering a correct formula, see Note 1 in the 'Notes' section at the end of these instructions.

Once you are happy that your calculation is working correctly you can replicate it down the whole column by dragging the small black handle at the lower right-hand corner of the cell containing the formula.

Calculate uncertainty in distance

The calculations of distances are based on parallax data that are known to an accuracy of ±1 milli-arcsec. Having calculated the distances it is now necessary to calculate the *uncertainty*.

How is the uncertainty in the calculated distance related to the ±1 milli-arcsec uncertainty in the parallax measurements? For this calculation the appropriate rule is that for a quantity x, the *relative* uncertainty in $y = 1/x$ is the same as the relative uncertainty in x (so if x is accurate to 10%, y is also accurate to 10%). (See Box 1.)

Box 1 Relative and absolute uncertainties

Whenever a measured quantity is reported it is very important to give an indication of the *accuracy* of the result. This is done by giving both the measurement and the uncertainty in the measurement.

If a quantity A has been measured to an accuracy of ΔA, then the result would be quoted as: $A \pm \Delta A$. The quantity ΔA is known as the *absolute uncertainty*.

The absolute uncertainty does not tell the whole story, however. Clearly a value of 100 ± 1 represents a higher accuracy than a value of 10 ± 1. It is therefore often useful to express the uncertainty as a *relative* quantity – for a measurement of $A \pm \Delta A$ the *relative uncertainty* is the fraction $\Delta A/A$.

Relative uncertainties are frequently expressed as *percentages*, so for a quantity ($A \pm \Delta A$) of 10 ± 1, the relative uncertainty $\Delta A/A$ would be 1/10, or 10%.

In this case, the distance d is calculated as the reciprocal of the parallax p.

The *relative uncertainty* in d is thus the same as the relative uncertainty in p:

$$\frac{\Delta d}{d} = \frac{\Delta p}{p}$$

In order to calculate the *absolute* uncertainty Δd, this equation can be rearranged as:

$$\Delta d = d\frac{\Delta p}{p}$$

giving a simple formula that can be used in the spreadsheet.

Notice that doing it this way saves a lot of unnecessary arithmetic: p and Δp can be left in units of milli-arcsec, and there is no need to convert $\Delta p/p$ into a percentage and then back again.

The formula gives Δd directly. You should always try to reduce your calculations to a simple formula before starting to work with the actual numbers.

Click on the cell that will hold the result of the calculation of uncertainty in distance of the first star in the list, (in Figure 3 this is cell D10).

Now enter the appropriate formula for the distance uncertainty calculation. If you are having difficulties in doing this see Note 2 in the 'Notes' section found towards the end of this activity.

Check this formula on the data for the first star before replicating it down the column.

Question 1

What do you notice about the uncertainties in the distances? What implications does this have for using the parallax method of determining stellar distances?

At first sight, it might appear that all that is needed to extend the range of the method is to increase the accuracy of the angular measurements of parallax. However, because of atmospheric blur ('seeing'), stars appear as a blur of finite size rather than an absolute point if measurements are conducted from the ground. Also, the actual diameter of the star may be larger than the parallax movement we are trying to measure (Betelgeuse has a diameter of 50 milli-arcsec). As you can imagine, it is very difficult to measure movements very much smaller than the diameter of the star's image. Even for telescopes mounted on spacecraft, like Hipparcos, there is a theoretical limit to the size of an image that is seen for a point-like source, which is related to the diameter of the telescope; the larger the telescope, the smaller the image that can, in theory, be produced. This theoretical limit will be reached only if the surfaces of the mirrors and/or lenses in the telescope are accurate to a fraction of the size of a wavelength of light and the stability of the pointing of the telescope is precise. For Hipparcos this theoretical limit to the image size is around 400 milli-arcsec, so the accuracy to which the position of the centre of such a star's image can be determined is quite remarkable. It is equivalent to the width of a golf ball viewed across the Atlantic Ocean!

Calculate transverse speed

In order to calculate the transverse speed from the distance and proper motion a little geometry is required:

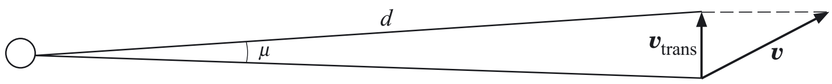


Figure 4 Geometry of stellar motion.

Since the angle μ is very small, the formula is apparently quite straightforward:

$$v_{\text{trans}} = d\mu$$

But once again you will need to think carefully about *units*.

■ The values that you have for d and μ so far are given in parsecs and milli-arcsec per year respectively. If you want to obtain an answer for v_{trans} in km s^{-1}, what units for d and μ do you need to use in the formula $v_{\text{trans}} = d\mu$? (You may want to refer to *An Introduction to the Sun and Stars* Section 3.2.1).

❑ For the calculation to give a result in km s^{-1}, the distance d must be expressed in km and the proper motion μ in radians s^{-1}. You will therefore need to *convert* the values into the correct units. (See Box 2.)

Box 2 Unit conversions

The following conversion factors are useful for the calculations required in this activity

1 radian = 180/π degrees = (180/π) × 3600 arcsec = 206 265 arcsec
= 2.06×10^8 milli-arcsec

1 milli-arcsec = $1/(2.06 \times 10^8) = 4.8 \times 10^{-9}$ radians

1 year = 365.25 days = 365.25 × 24 hours = 365.25 × 24 × 3 600 s
= 3.16×10^7 s

1 milli-arcsec $yr^{-1} = (4.8 \times 10^{-9}$ radians)/(3.16×10^7 seconds)
= 1.54×10^{-16} radians s^{-1}

1 parsec = 3.09×10^{16} m = 3.09×10^{13} km

Question 2

The spreadsheet gives distance in parsecs and proper motion in terms of milli-arcsec per year. What are the relevant conversion factors to (a) express the distance in kilometres, and (b) express the proper motion in rad s^{-1}?

You now need to enter a formula for the transverse speed of the first star in the list (in Figure 3, this is cell F10). Take care to include the conversion factors discussed in Question 2. You may find it useful to know that numbers in scientific notation can be input into a spreadsheet as shown in the examples in Table 2.

Table 2 Examples of how numbers in scientific notation are represented in a spreadsheet.

Number	Spreadsheet
3.84×10^{26}	3.84E+26
1.602×10^{-19}	1.602E-19

As in the previous calculations, you should check your result before replicating the formula for all seven stars. If you are having difficulties in obtaining or entering the correct formula see Note 3 in the 'Notes' section.

Appendix B to this activity shows how you can use named references to help clarify the formulae that you use in spreadsheet cells. This technique can be used in the formula that you have used for the transverse speed, so if you would like to try out using this facility it is recommended that you read Appendix B now. If, however, you are new to using spreadsheets it is probably best to skip Appendix B on your first reading of this activity.

Again, it is good practice to save the spreadsheet at this point.

Calculate uncertainty in transverse speed

The remaining quantity to be calculated is the uncertainty in the transverse speed. This is a bit more involved than the calculations that you have already done, and

this might be an appropriate place to take a break from this activity. If you do, make sure to save your spreadsheet before closing the StarOffice program.

The transverse speed is found by multiplying together two quantities, d and μ which both have associated uncertainties. In the following section (and Box 3), we'll look at how we handle uncertainties that are formed by the combination of two quantities.

Box 3 Combining uncertainties

When two quantities are added or subtracted the absolute uncertainties must be combined. This is not as simple as adding or subtracting the uncertainties.

Adding or subtracting quantities:

If you have two quantities $(x \pm \Delta x)$ and $(y \pm \Delta y)$ then the absolute uncertainty in $(x + y)$ is given by: $\sqrt{\Delta x^2 + \Delta y^2}$. The uncertainty in $(x - y)$ is the same, so be careful when subtracting quantities!

This process of squaring and adding is known as combining the uncertainties *in quadrature*. It gives a smaller result than simply adding the uncertainties, and thus takes into account the random nature of uncertainties in the two quantities x and y, which are independent of each other.

A simple way of understanding this is to consider an example: x and y are measurements of the length of two blocks of wood using a mm ruler. Each has an *estimated* uncertainty, Δx and Δy respectively. What this is really saying is that the true answer for the length of x is *likely* to lie in the range from $(x - \Delta x)$ to $(x + \Delta x)$. The measured value may be smaller or larger than the true value and the probability of it being Δx or more from the true value is small. Likewise, for the length y and its estimated uncertainty Δy. If we add these quantities (i.e. $z = x + y$) then we have the combined length of both blocks. However, it is *very* unlikely that the measured values of x and y were *both* too large (or both too small) by amounts Δx and Δy, as implied by assuming $\Delta z = \Delta x + \Delta y$. The range of likely values of z will be rather smaller than from $(z - \Delta x - \Delta y)$ to $(z + \Delta x + \Delta y)$. We therefore combine the quantities in quadrature to reflect the probable range of values.

Multiplying or dividing quantities:

When two quantities are multiplied or divided, you need to combine the *relative* uncertainties.

The relative uncertainty in $z = (x \times y)$ [or $(x \div y)$] is given (combining the relative errors in quadrature) by:

$$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

Question 3

Write down the equation for the absolute uncertainty in transverse speed in terms of the relative uncertainty in distance and proper motion.

Your task now is to insert a formula for the uncertainty in transverse speed of the first star in the list (Sirius) in the relevant cell. (In the spreadsheet of Figure 3, this would be cell G10.)

The equation that you obtained in Question 3 is somewhat more complicated than those that you have used so far, so we'll guide you through the process of writing the corresponding spreadsheet formula. In the following we will assume that the spreadsheet has the row and column layout as shown in Figure 3 and give the appropriate cell references.

The uncertainty in transverse speed will be held in cell G10. The equation we want to use is (see Box 3 and the answer to Question 3)

$$\Delta \upsilon_{\text{trans}} = \upsilon_{\text{trans}} \sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta \mu}{\mu}\right)^2} \tag{1}$$

- ■ For the first star in the list (Sirius) what are the cell references of υ_{trans}, d, Δd and μ?
- ❑ The cell references are:

  υ_{trans} is in cell F10, d is in cell C10, Δd is in cell D10, μ is in cell E10.

Thus we have cell references for all of the terms that appear on the right-hand side of Equation 1 with the exception of $\Delta\mu$. However a value of $\Delta\mu$ is given in the note on Table 1 – the proper motion has an uncertainty of ±1 milli-arcsec yr^{-1}.

So all of the variables that appear on the right-hand side of Equation 1 are known. The task now is to write a spreadsheet formula that represents the mathematical relationship between these variables. To do this you need to know the following spreadsheet conventions:

The square of a number is calculated using the operation '^2'. So, for instance, a cell that calculates the square of the quantity in cell B3 would be written as `=B3^2`.

The square root of a number is found by using the operation 'SQRT()': this returns the square root of the number inside the brackets. For example, a cell that calculates the square root of the quantity in cell F10 would be written as `=SQRT(F10)`.

Question 4

Write down the formula that calculates the uncertainty in transverse speed using the cell references given above.

Now that you have the formula for the uncertainty in the transverse speed insert it into the appropriate cell on your spreadsheet. As before, check the result is correct before replicating the cell for all stars.

You have now obtained results for distance and transverse speed in your spreadsheet. The data in the spreadsheet may, however, not have the correct number of significant figures. Although it is possible to adjust the number of decimal places displayed in any given cell on a spreadsheet, the correct number to use is not apparent until you have completed the uncertainty calculations.

Question 5

Rather than altering the formatting of each cell, which is very time consuming, finish off the activity by recording your final results in Table 3 below. Make sure that you quote your results to an appropriate number of significant figures.

Table 3 An empty table in which to record the results of the activity. For use with Question 5.

Star name	**Distance /pc**	**Uncertainty in distance/pc**	**Transverse speed /km s^{-1}**	**Uncertainty in transverse speed/km s^{-1}**
Alpha CMa (Sirius)				
Alpha Ori (Betelgeuse)				
Beta Ori (Rigel)				
Alpha Tau (Aldebaran)				
Alpha UMa (Dubhe)				
Kapteyn's star				

Here are a couple of more tricky questions that relate to the data you have just derived.

Question 6

It is possible to increase the accuracy of one of the measured quantities (parallax or proper motion) if they are made with a ground-based telescope, without using a different telescope. State which quantity this is correct for and explain why.

Question 7

Would it be more worthwhile making further observations to obtain a better result for the transverse speed of (a) Betelgeuse, or (b) Rigel? Explain your answer.

Notes

Note 1

The equation that needs to be used is that which calculates distance from the parallax in milli-arcsec. The relationship between parallax and distance is

$$d/\text{pc} = \frac{1}{p/\text{arcsec}}$$ (*An Introduction to the Sun and Stars*, Equation 3.7)

In this case the data are given in terms of milli-arcsec, so the appropriate equation is

$$d/\text{pc} = \frac{1000}{p/\text{milli-arcsec}}$$

If the spreadsheet is laid out as shown in Figure 3, the parallax (p) of the first star in the list (Sirius) is in cell B10, and the formula for distance needs to be inserted into cell C10. The formula in cell C10 should be `=1000/B10`.

Note 2

The equation that relates the uncertainty in distance d to the uncertainty in parallax p is

$$\Delta d = d\frac{\Delta p}{p}$$

If the spreadsheet is laid out as shown in Figure 3, the parallax (p) of the first star in the list (Sirius) is in cell B10, the distance is in cell C10 and the formula for the uncertainty in distance needs to be inserted in cell D10. Since the uncertainty in parallax Δp has a numerical value of 1 milli-arcsec, the formula that should be inserted in cell D10 is `=C10*(1/B10)`. (This could be simplified to `=C10/B10`, but that has the disadvantage that it is not as easy to see the relationship between the spreadsheet equation and the algebraic equation.)

Note 3

The equation that relates transverse speed to proper motion and distance is

$$v_{\text{trans}} = d\mu$$

For this speed to be expressed in km s^{-1}, d must be given in km and μ in rad s^{-1}. In the spreadsheet however, we have a distance expressed in parsecs and a proper motion expressed in milli-arcsec per year. So these value have to be multiplied by conversion factors of 3.09×10^{13} and 1.54×10^{-16} respectively if they are to be used to calculate the speed in km s^{-1}. (See Question 2.)

If the spreadsheet is laid out as shown in Figure 1, the distance (d) of the first star in the list (Sirius) is in cell C10, the proper motion is in cell E10 and the formula for the transverse speed needs to be inserted in cell F10.

The formula that should be written in cell F10 is

```
=(C10*3.09E+13)*(E10*1.54E-16)
```

You could simplify this expression to `=4.76E-03*C10*E10`, but as in the case of the distance calculation, this would make it more difficult to see the logic behind your calculation and is not recommended.

Answers to questions

Question 1

The distances of the nearer stars are known quite accurately (Sirius to better than 1%), but the method becomes less accurate for greater distances. This is because the fixed accuracy limit of 1 milli-arcsec is applied to smaller angles, leading to an uncertainty of over 20% for Rigel. The parallax method thus has an upper limit of a few hundred parsecs if determined to ±1 milli-arcsec.

Question 2

(a) To convert a distance expressed in parsecs into kilometres it is necessary to multiply by a factor of 3.09×10^{13} km pc^{-1} (see Box 2).

(b) To convert a proper motion expressed in milli-arcsec yr^{-1} into rad s^{-1} it is necessary to multiply by a factor of 1.54×10^{-16} radians s^{-1} (see Box 2).

Question 3

Start by looking at the equation $\upsilon_{\text{trans}} = d\mu$. The two quantities are multiplied together. The rule for multiplication and division is to combine the *relative* uncertainties.

In this case, the relative uncertainty in transverse speed is given by:

$$\frac{\Delta\upsilon_{\text{trans}}}{\upsilon_{\text{trans}}} = \sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta\mu}{\mu}\right)^2}$$

so the absolute uncertainty in transverse speed is

$$\Delta\upsilon_{\text{trans}} = \upsilon_{\text{trans}}\sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta\mu}{\mu}\right)^2}$$

Question 4

The equation for uncertainty in transverse speed is

$$\Delta\upsilon_{\text{trans}} = \upsilon_{\text{trans}}\sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta\mu}{\mu}\right)^2} \qquad (1)$$

and the cell references or values of the variables on the right-hand side of this equation as shown in Table 4:

Table 4 The cell references or numerical values of variables from Equation 1.

Variable	υ_{trans}	Δd	d	$\Delta\mu$	μ
cell reference	F10	D10	C10	-	E10
numerical value	-	-	-	1	-

Thus the formula used to express the right hand side of Equation 1 is

```
=F10*SQRT((D10/C10)^2+(1/E10)^2)
```

Note that it is essential that the numbers of left- and right-hand brackets are equal.

Question 5

The final results, quoted to an appropriate number of significant figures are shown in Table 5.

Table 5 Summary of results (completed version of Table 3).

Star name	Distance /pc	Uncertainty in distance/pc	Transverse speed /km s^{-1}	Uncertainty in transverse speed/km s^{-1}
Alpha Cma (Sirius)	2.64	0.01	16.81	0.05
Alpha Ori (Betelgeuse)	132	17	18.4	2.5
Beta Ori (Rigel)	238	57	2.3	1.3
Alpha Tau (Aldebaran)	20.0	0.4	19.0	0.4
Alpha UMa (Dubhe)	37.9	1.4	25.4	1.0
Kapteyn's star	3.92	0.02	161.6	0.6

Question 6

The accuracy of the measurement of a position depends on the size of the telescope and the size of the 'blur' due to atmospheric seeing, so it is not possible to improve these using the same telescope. The apparent position of a star changes due to parallax, with the star tracking out a small ellipse on the sky over the course of a year. Observations need to be made approximately six months apart to observe the maximum apparent change in position. The component of the true motion of the star through space causes it to move across the sky with a given proper motion. If the time interval between observations is extended, this change in position will be larger and the proper motion can be determined more accurately.

Question 7

More observations could significantly improve the accuracy of the proper motion of Rigel but not the parallax (see answer to Question 6). The uncertainty in the transverse speed results from a combination of the relative uncertainties in the distance (i.e. parallax) and proper motion. Table 6 shows the values for Betelgeuse and Rigel.

Table 6 Relative uncertainties in distance and proper motion for Betelgeuse and Rigel.

	Betelgeuse	Rigel
relative uncertainty in distance, $\Delta d/d$	13%	3.4%
relative uncertainty in proper motion, $\Delta\mu/\mu$	24%	50%

For Betelgeuse, the larger relative uncertainty is in the parallax so improving the proper motion accuracy would not make a significant difference to the accuracy of the transverse speed. However, for Rigel the relative uncertainty in the proper motion is larger than for the parallax so the uncertainty in the transverse speed could be significantly reduced if observations were made for longer (unfortunately this cannot be done with Hipparcos as the satellite is no longer operational!).

Resources

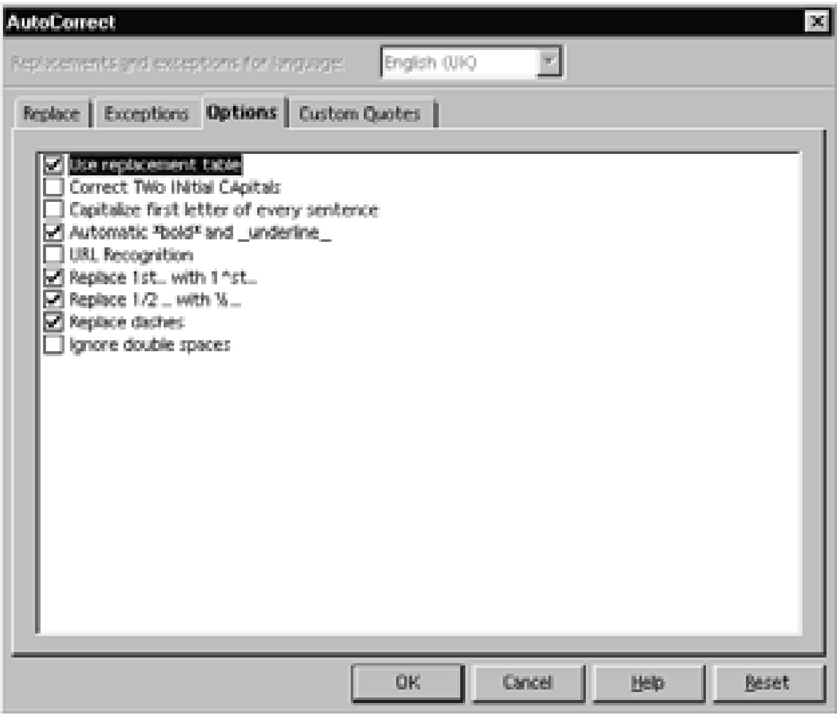
ESA Hipparcos catalogue http://www.rssd.esa.int/Hipparcos/

Appendix A Spreadsheet technique: autocorrect

Most commercial software for word processing and spreadsheets includes automatic checks for ‘mistakes’, such as ensuring that sentences start with a capital letter, or preventing more than one capital letter in a word. In addition, the software may recognize website or e-mail addresses and produce automatic links.

While these functions can be very useful, they may interfere with the way you want a document to look. The layout shown in Figure 3 is not possible without turning off some of these options (e.g. ‘Alpha CMa’ is automatically corrected to ‘Alpha Cma’ and ‘km s^-1’ is converted to ‘Km s^-1’). It is possible to switch off some of the automatic corrections in the StarOffice spreadsheet in the following way:

Select Autocorrect from the Tools menu and you will see the following box.



You can click on the small squares to toggle on or off any of the autocorrect functions. The screen shown here will allow you to reproduce exactly the spreadsheet layout shown in Figure 3.

Appendix B Spreadsheet technique: named constants

You have probably found out by now that formulae in spreadsheets can be quite tricky to write. Anything that can be done to clarify and make your formulae easier to understand will be useful to both yourself and to anyone reading or using the spreadsheet.

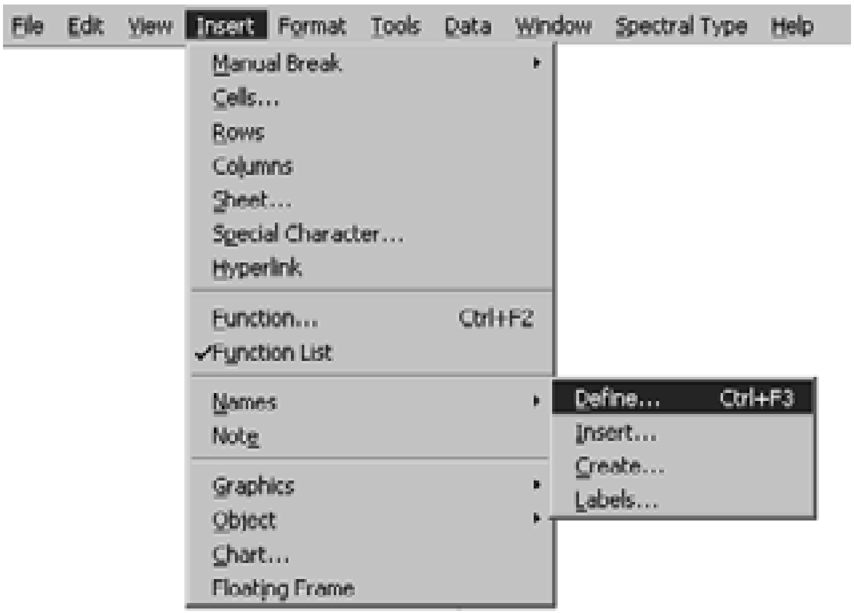
One way to improve the readability of your sheet is to use named constants instead of writing numbers or obscure cell references in your formulae.

Here, an area has been created to contain values for these conversion constants. Notice that each constant has been labelled in column A, and the units are noted in column B.

Now, you can give each cell containing a constant a name.

Highlight the cell containing the value that you want to give a name (cell B5 for the parsec to km conversion in this example).

From the StarOffice menu select Insert | Names | Define...



In the resulting dialog box, enter the name `_parsec` and press OK.

(You don't have to start the name with an underscore, but it avoids confusion with cell references and makes your formulae easier to follow. Other people using the spreadsheet will know that this is a named constant.)

You can now refer to this cell using either the reference B5 or the name _parsec.

So, for example, to convert a value (in parsecs) contained in cell C12 into km, you could enter the formula as follows: